DIFFERENTIAL EQUATIONS

Order & Degree of a differential equation

Equation	Order	Degree
$\left(\frac{d^2y}{dx^2}\right)^4 + \left(\frac{dy}{dx}\right)^5 - y = e^x$	2	4
$\frac{dy}{dx} = \frac{x^4 - y^4}{xy\left(x^2 + y^2\right)}$	1	1

- **Order** : Highest Derivative
- **Degree** : Power of Highest Derivative

Formation of a Differential Equation

- Differentiate the given equation w.r.t. the independent variable (say x) as many times as the number of arbitrary constants in it.
- Eliminate the arbitrary constants.

Solutions of Differential equation

- A relation between the dependent and independent variables (free from derivatives).

General Solution

- A relation in x and y satisfying a given differential equation and involving exactly the same number of arbitrary constants as the order of the diff. eqn.

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Particular Solution

- A solution obtained by assigning values to one or more than one arbitrary constant of general solution.

Variable Separable form

- If a differential equation can be expressed in the form $f(x)\,dx + g(y)\,dy = 0$, it can be integrated seperately.

Reducible to Variable Separable form

- If eqn of the form $(dy/dx) = f(ax + by + c)$,
  - put $ax + by + c = t$
- **Parametric form I**, $x = r\cos\theta$, $y = r\sin\theta \Rightarrow x^2 + y^2 = r^2$
  - $xdx + ydy = rdr$; $xdy - ydx = r^2 d\theta$
- **Parametric form II :** $x = r\sec\theta$, $y = r\tan\theta \Rightarrow x^2 - y^2 = r^2$
  - $xdx - ydy = rdr$; $xdy - ydx = r^2\sec\theta d\theta$
- $f(x, y)$ is said to be a **homogeneous expression** of degree n in the variables x and y if $f(tx,ty) = t^n f(x,y)$

If $\dfrac{dy}{dx} = \dfrac{f(x,y)}{g(x,y)}$ then $y = vx \Rightarrow \displaystyle\int \frac{dx}{x} = \int \frac{dv}{f(v) - v} + c$

Linear Differential Equation

- if the dependent variable (y) and its derivative appear only in the first degree

$$\frac{dy}{dx} + Py = Q \text{ then } ye^{\int P dx} = \int Qe^{\int P dx} + c$$

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Reducible to Linear form

Bernoulli Equation $\frac{dy}{dx} + Py = Qy^n$

- Divide by y^n, then put $y^{1-n} = t$, we get Linear differential equation in t and dt/dx.

Other Equation $\frac{dy}{dx} + P.f(y) = Q.g(y)$

- Divide by f(y), then put f(y)/g(y) = t and solve

Some of the Standard differentials to remember

$xdy + ydx = d(xy)$	$\frac{xdy - ydx}{x^2} = d\left(\frac{y}{x}\right)$
$\frac{ydx - xdy}{y^2} = d\left(\frac{x}{y}\right)$	$\frac{xdy + ydx}{xy} = d(\log xy)$
$\frac{dx + dy}{x + y} = d\log(x + y)$	$\frac{xdy - ydx}{xy} = d\left(\ell n\frac{y}{x}\right)$
$\frac{ydx - xdy}{xy} = d\left(\ell n\frac{x}{y}\right)$	$\frac{xdy - ydx}{x^2 + y^2} = d\left(\tan^{-2}\frac{y}{x}\right)$
$d\left(\frac{e^x}{y}\right) = \frac{ye^x dy - e^x dy}{y^2}$	

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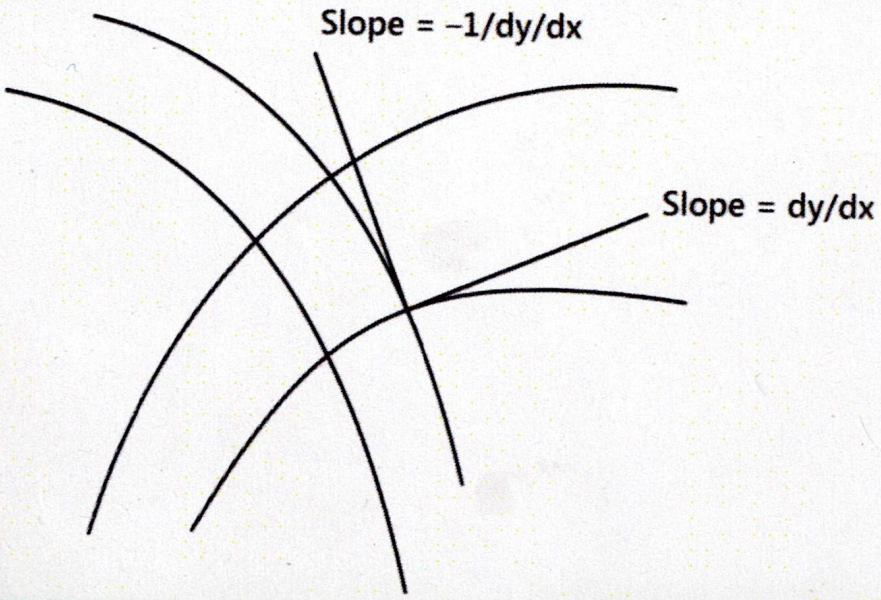
Orthogonal Trajectory

Two families of curves are such that each curve in either family is orthogonal (whenever they intersect) to every curve in the other family. Each family of curves is orthogonal trajectories of the other. In case the twofamilies are identical then we say that the family is self-orthogonal

How to Find Orthogonal Trajectories

1. Differentiate with respect to x
2. Eliminate the constant by putting the value in main equation
3. Change y′ to (−1/y′)
4. Integrate w.r.t y and x on different sides



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